For all questions, answer choice (E) NOTA means that none of the given answers is correct. The $\sqrt{-1} = i$. Good Luck!

1. What is the equation of a parabola with focus (2, 6) and directrix $y = 4$?
(A) $x = \frac{1}{4}y^2 - y + 6$ (B) $y = \frac{1}{16}x^2 - \frac{1}{4}x + 6$ (C) $x = \frac{1}{16}y^2 - \frac{1}{4}y + 6$ (D) $y = \frac{1}{4}x^2 - x + 6$ (E) NOTA

2. What is the inverse of the converse of the contrapositive of the statement, "If Tanusri is tired, then she drinks coffee."?
(A) "If Tanusri is tired, then she drinks coffee."
(B) "If she drinks coffee, then Tanusri is tired."
(C) "If Tanusri is not tired, then she does not drink coffee."
(D) "If she does not drink coffee, then Tanusri is not tired."
(E) NOTA

3. Let $f(x)$ be a cubic polynomial. Given that $f(-2) = 0, f(-1) = 3, f(0) = 10$ and $f(1) = 28$, what is the value of $f(2) + f(3)$?
(A) 47 (B) 50 (C) 125 (D) 189 (E) NOTA

4. Let $ABCD$ be a rectangle. A point P is chosen within the rectangle such that $AP = 2$, $BP = 5$, and $DP = 4$. What is the length of CP?
(A) $\sqrt{35}$ (B) 6 (C) $\sqrt{37}$ (D) $\sqrt{38}$ (E) NOTA

5. Rohan Da Vinci, bored in class, begins drawing on his paper. Suppose he draws a conic P that has eccentricity 0 and area 72π. Akash, also bored in class, draws three distinct points A, B, and C on the conic such that $\angle ABC = 90^\circ$. Find the sum of the digits of AC^2.
(A) 9 (B) 11 (C) 12 (D) 18 (E) NOTA

6. What is the coefficient of x^9 in the expansion of $\left(\frac{x^3}{2} - \frac{3}{x^4}\right)^{10}$?
(A) $-\frac{15}{16}$ (B) $-\frac{27}{128}$ (C) $\frac{27}{1024}$ (D) $\frac{15}{128}$ (E) NOTA

7. Let the equation of M be $x^2 + y^2 + 6x - 10y = 66$. Given that point A represents the center of M, and points B and C represent the intersection points of $x - y = -18$ and M, what is the area of triangle ABC?
(A) 50 (B) $50\sqrt{2}$ (C) 100 (D) $100\sqrt{2}$ (E) NOTA

8. Which of the following is an asymptote of $4y^2 - 2y + 7x - 7x^2 = \frac{25}{4}$?
(A) $x - \frac{1}{4} = \frac{\sqrt{7}}{2}(y - \frac{1}{2})$ (B) $y - \frac{1}{4} = \frac{\sqrt{7}}{2}(x - \frac{1}{2})$ (C) $x - \frac{1}{4} = \frac{\sqrt{5}}{2}(y - \frac{1}{2})$ (D) $y - \frac{1}{4} = \frac{\sqrt{5}}{2}(x - \frac{1}{2})$ (E) NOTA

9. What is the sum of the real solutions of $e^{4x} - 4e^{2x} + 6 = 3$?
(A) no real solutions (B) $\ln 3$ (C) $\ln 6$ (D) $\ln \sqrt{3}$ (E) NOTA

10. Let the roots of polynomial $g(x)$ be $10, -1$, and 9, and all roots have multiplicity 1. If $g(1) = 36$, what is the value of $g(3) - g(2)$?
(A) 0 (B) 6 (C) 54 (D) 90 (E) NOTA

11. Evaluate $\sum_{n=6}^{\infty} \frac{1}{2n^2 - 16n + 30}$.
(A) $\frac{3}{8}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1 (E) NOTA

12. Karthik, Tanmay, Eric, and Shubham are having a competition to see whose 3-D shape has the greatest volume. Karthik has a rectangular prism with faces of areas 54, 24, and 36. Tanmay has a frustrum with a height of 3 and radii equivalent to the roots of $x^2 - 14x + 48 = 0$. Eric has a regular tetrahedron with edge length 6. Shubham has a cube with a space diagonal length of 12. Who is the winner of the competition?
(A) Karthik (B) Tanmay (C) Eric (D) Shubham (E) NOTA

13. Let $Q(x)$ be a polynomial that has a remainder of 3 when divided by $x - 4$ and a remainder of 6 when divided by $x - 7$. What is the remainder when $Q(x)$ is divided by $(x - 4)(x - 7)$?
(A) $x - 28$ (B) $x - 14$ (C) $x - 11$ (D) $x - 1$ (E) NOTA

14. Sina the Beekeeper notices that his special bees leave behind a trail of honey in the form of a hexagon with area $108\sqrt{3}$ cm^2. What is the sum of the areas of the circle inscribed and the circle circumscribed about the hexagon?
(A) 108π (B) 126π (C) 216π (D) 252π (E) NOTA

15. Find the sum of the cubes of the roots of $f(x) = x^3 + x + 10$.
(A) -30 (B) -10 (C) 0 (D) 30 (E) NOTA

16. In $\triangle ABC$, points D and E lie on BC and AB, respectively. AD and CE intersect at point F such that $AF : FD = 3 : 2$ and $CF : FE = 4 : 1$. What is the value of $BD : DC$?
(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 1 (E) NOTA

17. $\triangle ABC$ has side lengths 13, 14, and 15. If the center of the incircle of ABC is $(5, -3)$, which of the following points will not lie on the circumference of the incircle?
(A) (1, 0) (B) $(5 - \sqrt{15}, -2)$ (C) $(3, 2\sqrt{3} + 3)$ (D) $(0, -7)$ (E) NOTA

18. In $\triangle ABC$, a cevian from point B intersects side AC at D. If $AB = 12$, $BC = 14$, $BD = 6$, and $AD = 3 \cdot DC$, then what is the value of AC?
(A) 10 (B) $2\sqrt{13}$ (C) $8\sqrt{6}$ (D) 28 (E) NOTA

19. Given cyclic quadrilateral $ABCD$, let $AB = 5$, $BC = 12$, $CD = 10$, and $AD = 2$. What is the value of $2 \cdot AC$?
(A) 16 (B) $\sqrt{286}$ (C) $\sqrt{290}$ (D) 18 (E) NOTA

20. What is the value of $\sin(\arccos(\frac{9}{15})) + \tan(\arccos(\frac{9}{15}))$?
(A) $\frac{12}{9}$ (B) $\frac{7}{5}$ (C) $\frac{15}{9}$ (D) $\frac{32}{15}$ (E) NOTA

21. The quantity $\log 1^1 + \log 2^2 + \log 3^3 + \cdots + \log 100^{100}$ can be expressed as

$$a_2 \log 2 + a_3 \log 3 + a_5 \log 5 + \cdots + a_{97} \log 97$$

with unique positive integers $(a_2, a_3, \cdots a_{97})$. Find $a_7 + a_{13}$.
(A) 1200 (B) 1225 (C) 1234 (D) 1246 (E) NOTA

22. Let r and s be the roots of $x^2 + 7x - 10 = 0$. Evaluate

$$\frac{r^2 + 1}{1 - r} + \frac{s^2 + 1}{1 - s}.$$

(A) -4 (B) $\frac{7}{10}$ (C) 3 (D) Cannot be found (E) NOTA

23. When graphed on a Cartesian plane, one vertex of an equilateral triangle is (3, 4). The other two vertices both fall on the line $x + y\sqrt{3} = 1$. What is the perimeter of the triangle?
(A) 12 (B) $\frac{12+4\sqrt{3}}{5}$ (C) 16 (D) $\frac{16+4\sqrt{3}}{5}$ (E) NOTA

24. If $\frac{1}{2} + \frac{2}{6} + \frac{5}{36} + \frac{1}{4} + \frac{8}{216} + \frac{11}{1296} + \frac{1}{8} + \ldots = \frac{m}{n}$, where m and n are relatively prime positive integers, what is $m + n$?
(A) 23 (B) 38 (C) 53 (D) 63 (E) NOTA

25. Evaluate the expression
$$\log_{10}\left(\sum_{a_1=1}^{10}\sum_{a_2=1}^{10}\sum_{a_3=1}^{10}\cdots\sum_{a_{10}=1}^{10} 0.000001\right).$$
(A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

26. Given that none of x, y, z is divisible by 3, how many nonnegative integer solutions are there to $x + y + z = 60$?
(A) 200 (B) 300 (C) 400 (D) 600 (E) NOTA

27. What is the sum of the real solutions of $(5 - 2\sqrt{6})^{(x^2-6x+5)} + (5 + 2\sqrt{6})^{(x^2-6x+5)} = 10$?
(A) $3 + \sqrt{5}$ (B) 6 (C) 12 (D) $3 - \sqrt{3}$ (E) NOTA

28. Find $ABC - D$ given that
$$\begin{vmatrix} x+2 & x & 1 \\ 0 & x-5 & x \\ 2x-5 & 0 & 3x-1 \end{vmatrix} = (x - A)(Bx^2 + Cx + D) - 69.$$
is a true statement for all real numbers x, and A, B, C, D are integers.
(A) -48 (B) -12 (C) 0 (D) 24 (E) NOTA

29. How many ordered pairs (x, y, z) are there such that each is a positive integer between 1 and 7 (inclusive) and the product xyz is divisible by 10?
(A) 27 (B) 63 (C) 90 (D) 100 (E) NOTA

30. Simplify $\frac{1}{\sqrt{2}+\sqrt{7}-\sqrt{5}}$.
(A) $\frac{\sqrt{70}-5\sqrt{2}-2\sqrt{5}}{20}$ (B) $\frac{\sqrt{70}-5\sqrt{2}+2\sqrt{5}}{20}$ (C) $\frac{\sqrt{70}+5\sqrt{2}-2\sqrt{5}}{20}$ (D) $\frac{\sqrt{70}+5\sqrt{2}+2\sqrt{5}}{20}$ (E) NOTA